Using a Quadratic Function to Model Vertical Motion

HABITS OF MIND
- Model with mathematics.
- Use appropriate tools strategically.

You can model the motion of a pumpkin released from a catapult using a vertical motion model. A **vertical motion model** is a quadratic equation that models the height of an object at a given time.

> Consider the equation for a vertical motion model.

$$y = -16t^2 + v_0 t + h_0$$

In this equation, y represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

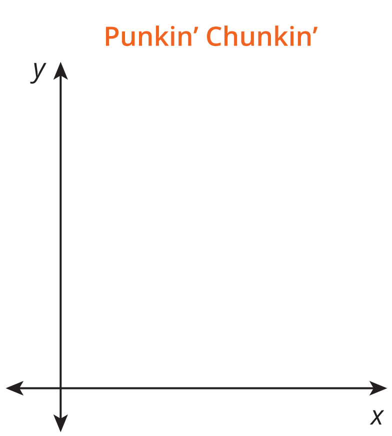
1. Which characteristics of this situation indicate that you can model it using a quadratic function?

Suppose that a catapult hurls a pumpkin from a height of 68 feet at an initial vertical velocity of 128 feet per second.

2. Write a function for the height of the pumpkin, $h(t)$, in terms of time, t.

3. Does the function you wrote have a minimum or maximum? **How can you tell from the form of the function?**

4. Use technology to graph the function. Sketch your graph and label the axes.



ASK YOURSELF . . .
What do all the points on this graph represent?

5 Use technology to determine the maximum or minimum point and label it on the graph. **Explain what it means in terms of the problem situation.**

6 Determine the y-intercept and label it on the graph. **Explain what it means in terms of the problem situation.**

7 Use a horizontal line to determine when the pumpkin reaches each height after being catapulted. Label the points on the graph.

(a) 128 feet

(b) 260 feet

(c) 55 feet

8 Explain why the x- and y-coordinates of the points where the graph and each horizontal line intersects are solutions.

9 When does the catapulted pumpkin hit the ground? Label this point on the graph. **Explain how you determined your answer.**

The time when the pumpkin hits the ground is one of the x-intercepts, $(x, 0)$. When you use an equation to model a situation, you refer to the x-coordinate of the x-intercept as the *root*. The **root** of an equation indicates where the graph of the equation crosses the x-axis.

REMEMBER . . .
The zeros of a function are the x-values when the function equals 0.

ACTIVITY 3

SUMMARY You call the x-coordinate of each x-intercept of a quadratic equation a root. The roots of an equation indicate the x-values when $y = 0$.

ACTIVITY 3

MATHia CONNECTION
- Modeling Projectile Motion
- Recognizing Key Features of Vertical Motion Graphs

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> Consider the equation for a vertical motion model.

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In this equation, y represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

1. Which characteristics of this situation indicate that you can model it using a quadratic function?

Sample answer:
If I catapult a pumpkin in the air, its height increases and then decreases over time, and it does not travel at the same rate the entire time. So, it makes sense to model this situation using a parabola.

Suppose that a catapult hurls a pumpkin from a height of 68 feet at an initial vertical velocity of 128 feet per second.

2. Write a function for the height of the pumpkin, $h(t)$, in terms of time, t.

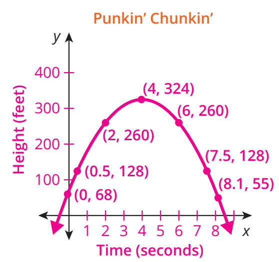
$h(t) = -16t^2 + 128t + 68$

3. Does the function you wrote have a minimum or maximum? **How can you tell from the form of the function?**

Because the coefficient of x^2 is negative, the function increases then decreases. Therefore, it has a maximum.

4. Use technology to graph the function. Sketch your graph and label the axes.

ASK YOURSELF . . .
What do all the points on this graph represent?



Chunking the Activity

- Read and discuss the introduction
- Group students to complete 1
- Check in and share
- Group students to complete 2–4
- Check in and share
- Complete 5 as a class
- Group students to complete 6–9
- Share and summarize

DIFFERENTIATION STRATEGY
See page 672B to assist all students with 4.

COMMON MISCONCEPTION
See page 672B for a misconception related to 4.

LANGUAGE LINK

ELL TIP

Ensure students understand each of the situations by defining terms such as *catapult*.

Questions to Support Discourse

		TYPE
Intro	• What is meant by *vertical motion*? • Why do you think there is an initial velocity? Will the speed of the pumpkin change?	Gathering
2	• Explain how you created the function for this situation. • How is this equation the same and different from those you wrote for the dog enclosure and handshake problems?	Probing
4	• Is the parabolic shape the actual path of the pumpkin in the air? Why or why not? • If the function models the up/down or vertical motion, why is the graph ∩-shaped? What does the x-axis label have to do with this?	Probing

NOTES

ACTIVITY 3 Continued

5 Use technology to determine the maximum or minimum point and label it on the graph. **Explain what it means in terms of the problem situation.**

The maximum point is (4, 324).

It means that the pumpkin reached a maximum height at 324 feet after 4 seconds.

6 Determine the y-intercept and label it on the graph. **Explain what it means in terms of the problem situation.**

The y-intercept is (0, 68).

The initial height of the pumpkin at 0 seconds was 68 feet.

7 Use a horizontal line to determine when the pumpkin reaches each height after being catapulted. Label the points on the graph. **See graph in Question 4.**

(a) 128 feet

The pumpkin is at a height of 128 feet 0.5 second and 7.5 seconds after being catapulted.

(b) 260 feet

The pumpkin is at a height of 260 feet 2 seconds and 6 seconds after being catapulted.

(c) 55 feet

The pumpkin is at a height of 55 feet 8.1 seconds after being catapulted.

8 Explain why the x- and y-coordinates of the points where the graph and each horizontal line intersects are solutions.

The graph of the parabola represents all the possible times and corresponding heights of the pumpkin. The graph of each horizontal line represents a specific height. So, the intersection between those two lines represents the specific time, or x-coordinate, for the specific height.

9 When does the catapulted pumpkin hit the ground? Label this point on the graph. **Explain how you determined your answer.**

The pumpkin hit the ground 8.5 seconds after being catapulted.

Sample answer:
I used the zero feature on my graphing calculator to determine the answer.

The time when the pumpkin hits the ground is one of the x-intercepts, (x, 0). When you use an equation to model a situation, you refer to the x-coordinate of the x-intercept as the *root*. The **root** of an equation indicates where the graph of the equation crosses the x-axis.

REMEMBER . . .
The zeros of a function are the x-values when the function equals 0.

666 Topic 1 > Introduction to Quadratic Functions

Questions to Support Discourse

		TYPE
6	• Where does the y-intercept appear in the function?	Seeing structure
7	• How did you use the horizontal line to solve this problem? • How can the pumpkin be at the same height at two different times? • Why is there only one possible time for 55 feet when there are two possible times for the other heights?	Probing
9	• What y-value represents the height at ground level?	Gathering
	• Why didn't you include the time at the other x-intercept?	Probing
	• Do you think a quadratic equation can have more than one root? Explain your thinking.	Seeing structure

ACTIVITY 3

Session 2 of 2

Using a Quadratic Equation to Model Vertical Motion

Students use a situation involving vertical motion to explore the roots of quadratic equations. They use technology to graph the function and solve the function for specific *x*-values. They identify and interpret the maximum point, *y*-intercept, and *x*-intercepts in terms of the situation.

CHUNK	AUDIENCE	ADDITIONAL SUPPORTS
As students work on 4	All students	**DIFFERENTIATION STRATEGY** Establish what should be visible and labeled as students sketch the graph of a parabola: the *x*-intercepts, *y*-intercept, and maximum/minimum point.
As students work on 4	Students who struggle	**COMMON MISCONCEPTION** Students may interpret the graph as the path of the pumpkin. Discuss that the graph relates time and height, not horizontal distance and height. Demonstrate this difference by tossing a ball vertically. The graph modeling the ball's height over time is ∩-shaped even though the ball goes straight up and down.